ALTAM Fall 2025

Model Solutions and Examiners' Comments

These solutions and examiners' comments are intended to be used by candidates preparing for future ALTAM exams. In many places they are more detailed than would be required for full credit under examination conditions. In some cases there also be valid alternative methods or derivations that are not included in these solutions, but that were given appropriate credit in grading.

Question 1

See Excel workbook.

Question 2 (*7 points*)

(a) (*1 point*) Type A policies offer a level death benefit equal to the specified face value (unless corridor factors apply).

Type B policies offer a death benefit equal to a specified level additional death benefit plus the account value, so the total death benefit increases with the account value.

(b) (*2 points*) Because there are no corridor factors, we have

$$AV_1 = \left((1500)(0.97) - 50 - 1.2\, q_{40}\, v_{i_q}\left(100{,}000 - AV_1\right)\right)(1.04)$$

$$\Rightarrow \text{AV}_1 = \frac{\left((1500)(0.97) - 50 - 1.2\, q_{40}\, v_{i_q}(100{,}000)\right)(1.04)}{1 - 1.2\, q_{40}\, v_{i_q}(1.04)} = \mathbf{1{,}397.62}$$

(c) (*2 points*)

(i) At time 20:

$$V^{nlg} = \max\left(100{,}000 A^{1}_{60:\overline{30}|} - AV_{20}, 0\right)$$

$$A^{1}_{60:\overline{30}|} = A_{60} - v^{30}\frac{l_{90}}{l_{60}}A_{90} = 0.21483$$

$$\Rightarrow V^{nlg} = \max(21{,}483 - 33{,}590, 0) = \mathbf{0}$$

(ii) At time 40:

$$V^{nlg} = \max\left(100{,}000 A^{1}_{80:\overline{10}|} - AV_{40}, 0\right)$$

$$A^{1}_{80:\overline{10}|} = A_{80:\overline{10}|} - {}_{10}E_{80} = 0.33722$$

$$\Rightarrow V^{nlg} = \max(33{,}722 - 26{,}403, 0) = \mathbf{7{,}319}$$

(d) (*2 points*) Advantages:

(i) Offering the guarantee might offer a competitive advantage over other insurers and result in more sales.

(ii) Requiring a minimum premium level for the guarantee to be valid could entice policyholders to pay more in premiums than they otherwise would have, which will tend to make the product more profitable for the insurer.

Disadvantages:

(i) If the guarantee ends up being “in the money”, there could be significant costs for the insurer, especially since this risk is not fully diversifiable.

(ii) The insurer should hold reserves for the guarantee, which is potentially an additional cost arising late in the policy term.

Examiners' comments

1. *Parts (a) and (b) were done well by most candidates.*
2. *Part (c) proved much more challenging. Only around 15% of candidates knew how to calculate the reserves. The no-lapse guarantee is a topic that is in the sample questions and is explained in the text book, and the calculation is fairly straightforward, so the poor performance on this part was unexpected.*
3. *Part (d) was done well by the candidates who knew what a no-lapse guarantee is. Good candidates explained how premium persistency could improve profits, and they also knew the risks to the insurance company. Several candidates had a fundamental misunderstanding of NLGs, conflating them with short-pay options.*

Question 3 (*11 points*)

(a) *(4 points)*

(i) First suppose that at time t, both (x) and (y) are alive. The marginal force of mortality for (x) at time t is then $\mu^{02}_{x+t:y+t} + \mu^{04}_{x+t:y+t} = \mu^{s}_{x+t}$.

Similarly, if (y) has died by time t, and (x) has survived, the marginal force of mortality for (x) at time t is $\mu^{13}_{x+t} = \mu^{s}_{x+t}$.

Since the force of mortality function determines the distribution of T_x, and since the marginal force of mortality for (x) is the same as the SUSM force of mortality regardless of the status of (y), then the marginal future lifetime random variable, T_x, follows the SUSM distribution.

(ii) The lives are not independent because they are subject as a couple to common shock risk. That means that information about the survival status of one life gives information about the survival status of the other.

Specifically, if we know that (x) survives t years, then we know the common shock event did not occur in those t years, which means that (y)'s survival probability is higher than it would be without that knowledge, because it tells us that (y)'s force of mortality over the same period is $\mu^{s}_{y+r} - \lambda$, which is lower than their marginal force of mortality.

(iii) There is no broken heart dependency as the force of mortality for each life is the same whether the other is alive or dead.

(b) (*2 points*)

$$
\begin{aligned}
{}_t p^{00}_{x:y} &= \exp\left\{-\int_0^t \mu^{01}_{x+r:y+r} + \mu^{02}_{x+r:y+r} + \mu^{04}_{x+r:y+r}\ dr\right\} \\
&= \exp\left\{-\int_0^t \left(\mu^{s}_{x+r} - \lambda\right) + \left(\mu^{s}_{y+r} - \lambda\right) + \lambda\ dr\right\} \\
&= \exp\left\{-\int_0^t \mu^{s}_{x+r} + \mu^{s}_{y+r} - \lambda\ dr\right\} = e^{-\int_0^t \mu^{s}_{x+r}\,dr} \times e^{-\int_0^t \mu^{s}_{y+r}\,dr} \times e^{\lambda t} \\
&= {}_t p^{s}_{x}\ {}_t p^{s}_{y}\ e^{\lambda t}
\end{aligned}
$$

(c) (*4 points*)

(i) If the lives die by common shock at time t, then the return of premium benefit is tP, so the EPV is

$$
\int_0^\infty (tP)\ {}_t p^{00}_{x:y}\ \mu^{04}_{x+t:y+t}\ v^t\ dt \quad = P\lambda \int_0^\infty t\, v^t\, {}_t p^{00}_{x:y}\ \ dt \quad = P\lambda \left(\bar{I}\bar{a}\right)^{00}_{x:y}
$$

:

(ii) We know that a unit reversionary annuity to (*x*) following the death of (*y*), payable continuously, has value $\bar{a}_x - \bar{a}^{00}_{x:y}$, and similarly for a reversionary annuity to (*y*). The single life annuities are from SUSM, and the joint life annuity value is given. Hence the EPV is

$$50{,}000\left(\left(\bar{a}_x - \bar{a}^{00}_{x:y}\right) + \left(\bar{a}_y - \bar{a}^{00}_{x:y}\right)\right) = 50{,}000\left(\bar{a}_{50} + \bar{a}_{55} - 2\bar{a}^{00}_{50:55}\right)$$

$$= 50{,}000\left(\left(17.0245 - 0.5\right) + \left(16.0599 - 0.5\right) - 2(14.7443)\right)$$

$$= \mathbf{129{,}790.08}$$

(iii) $P\bar{a}^{00}_{x:y} = 129{,}790.08 + 167.13\left(\lambda P\right)$

$$\Rightarrow P = \frac{129{,}790.08}{14.7443 - 0.0005 \times 167.13} \quad = \mathbf{8{,}852.90}$$

(d) (*1 point*) Let ${}_tV^{(j)}$ denote the net premium policy value at t given that the policy is in state j, for $j = 0,1,2$. Then

$$\frac{d}{dt}\,{}_tV^{(0)} = \delta\,{}_tV^{(0)} + P - \lambda\left(tP - {}_tV^{(0)}\right) - \mu^{01}_{y+t}\left({}_tV^{(1)} - {}_tV^{(0)}\right) - \mu^{02}_{x+t}\left({}_tV^{(2)} - {}_tV^{(0)}\right)$$

where ${}_tV^{(1)} = 50{,}000\,\bar{a}_x$ and ${}_tV^{(2)} = 50{,}000\,\bar{a}_y$

Examiners' comments

1. *Overall, candidates found this question quite challenging. It required a good understanding of how transition intensities in the joint life model combine to form marginal single life models, as well as an ability to coherently develop the associated functions to evaluate premiums and benefits.*
2. *Parts (a)(i) and (a)(ii) were done very well by most candidates. However, most candidates wrongly believed that the model incorporated a broken heart effect, or just described the broken heart effect without relating it to the model. In fact, as shown in (a)(i), the marginal mortality of (x) was unaffected by the death of (y), and vice versa. If there were a broken heart effect, then the mortality of (x) would be expected to increase on the death of (y), and/or vice versa.*
3. *In part (b), most candidates understood that they needed to integrate the sum of the transition forces, but a large number made unnecessary errors or unreasonable simplification in the derivation. In particular, many candidates treated the transition forces as if they were constant, either implicitly, by ignoring the integration variable in the subscript of the transition forces, or explicitly, by replacing the integrals with* $t\mu^s_x$ *and* $t\mu^s_y$. *Very little credit was given where the forces were explicitly assumed to be constant. Partial credit was awarded to candidates who omitted the integration variable in the formula, but who did not explicitly assume constant forces. For full credit, candidates were required to write the integrals correctly, including the integration variable in the subscripts.*

4. *Part (c)(i) was done well by a large number of candidates. Part (c)(ii) proved more challenging, but this was consistent with the examiners' expectations. Many candidates got part (c)(iii) correct by using the givens in (c) (i) and (c) (ii) even if they could not do those parts. This is an excellent strategy in answering these questions and one of the reasons that the Exam Committee provides the answers or a close approximation to the answers. Candidates using this approach correctly got full credit for (c) (iii).*
5. *Candidates who attempted part (d) generally did well. Around 30% of candidates did not attempt it.*

Question 4 (*10 points*)

(a) (*5 points*)

(i) If $t_j < 1$ and $\delta_j = 0$, then the *j*-th life was between ages *x* and *x*+1 for some period of the mortality study, but not for the whole year of age, and they did not die whilst in the study – either they entered observation after age *x* (e.g. bought a policy) , or they left observation, but did not die (e.g. surrendered their policy) before age *x*+1.

(ii) Let *L* denote the likelihood function, and let *l* denote the log-likelihood function.

$$L(\mu_x) = \prod_{j=1}^{n} f_{x_j}(t_j, \delta_j) \quad \Rightarrow \quad l(\mu_x) = \sum_{j=1}^{n} \log\left(f_{x,j}(t_j, \delta_j)\right) = \sum_{j=1}^{n} \left(-t_j \mu_x + \delta_j \log(\mu_x)\right)$$

$$\Rightarrow \quad l(\mu_x) = -\left(\sum_{j=1}^{n} t_j\right)\mu_x + \left(\sum_{j=1}^{n} \delta_j\right)\log(\mu_x)$$

(iii) We find the MLE by differentiating $l(\mu)$ and setting it equal to 0. Let $E_x^c = \sum_{j=1}^{n} t_j$ and let $d_x = \sum_{j=1}^{n} \delta_j$. Then we have

$$l(\mu_x) = -E_x^c \ \mu_x + d_x \log(\mu_x) \Rightarrow l'(\mu_x) = -E_x^c + \frac{d_x}{\mu_x}$$

$$\Rightarrow l'(\hat{\mu}_x) = 0 \Leftrightarrow \hat{\mu}_x = \frac{d_x}{E_x^c}$$

(iv) We estimate the asymptotic variance of the MLE by differentiating $l(\mu_x)$ twice, and using $Var[\hat{\mu}_x] \approx -\frac{1}{l''(\hat{\mu}_x)}$. From (a)(iii) we have

$$l'(\mu_x) = -E_x^c + \frac{d_x}{\mu_x} \Rightarrow l''(\mu_x) = -\frac{d_x}{\mu_x^2} \Rightarrow -\frac{1}{l''(\hat{\mu}_x)} = \frac{\hat{\mu}_x^2}{d_x}$$

$$\Rightarrow Var[\hat{\mu}_x] \approx \frac{d_x^2}{\left(E_x^c\right)^2} \times \frac{1}{d_x} = \frac{d_x}{\left(E_x^c\right)^2}$$

(b) (*1 point*) Let σ_A^2 denote the approximate variance of $\hat{\mu}_x^A$, and similarly for Line B. Then

$$\hat{\mu}_{65}^A = \frac{1550}{150150} = 0.01032; \quad \sigma_A^2 \approx \frac{1550}{150150^2} = 6.875 \times 10^{-8} = 0.00026^2$$

$$\hat{\mu}_{65}^B = \frac{132}{18070} = 0.00730; \quad \sigma_B^2 \approx \frac{132}{18070^2} = 4.043 \times 10^{-7} = 0.00064^2$$

(c) (*2 points*)

From asymptotic MLE properties, $\hat{\mu}_{65}^{A} - \hat{\mu}_{65}^{B}$ is approximately Normally distributed, with variance equal to the sum of the individual MLE variances.

We approximate this variance by $\sigma_A^2 + \sigma_B^2 = 4.3\times 10^{-7} = 0.00069^2$. Hence, a 95% CI for $\mu_{65}^{A} - \mu_{65}^{B}$ is $\left(\left(\hat{\mu}_x^A - \hat{\mu}_x^B\right) \pm 1.95(0.00069)\right) = (0.0017,\ 0.0044)$. As this CI does not contain 0, at the 95% confidence level the null hypothesis of equal forces is rejected.

(d) (*2 points*) Some possible answers:

1. Different business lines and self selection: We expect purchasers of life insurance to have heavier mortality than purchasers of annuities or other contracts with significant survivor benefits. Line A might be data from whole life insurance policyholders, and line B could be annuity purchasers.
2. Different underwriting standards – for example, Line B might be more strictly underwritten to include only ‘preferred lives’ i.e. lives with low mortality risk based on standard rating factors (smoking, health history, family health history), while Line A might be ‘Normal’ lives with higher mortality.
3. Different populations by geography – mortality varies by country and even by intra-national regions.
4. Different populations by socio-economic status. Higher income/socio-economic status is associated with lower mortality. So, for example, Line B could be more commonly purchased by higher income policyholders (perhaps an investment type product with a high minimum premium) compared with Line A, leading to lower mortality experience.

Examiners’ Comments

1. *Generally, candidates knew how to estimate forces of mortality from the data, but most were not able to derive the MLE.*
2. *Many candidates omitted this question entirely. This is an important part of the syllabus, and the derivations are relatively intuitive and are quite short.*
3. *In this question candidates were given the form of the individual joint probability function for the j-th life. In past exam papers, candidates were expected to know or construct this.*
4. *Some candidates lost points by not reading the question carefully, particularly in (a)(ii) and (iii). It is very good practice in this exam to read each question several times as you work through it, to ensure that you are answering the question asked as fully as possible, using the information given.*
5. *In part (a)(i), most candidates earned most of the credit, but many did not answer the full question. The question asked about a single individual with $t_j < 1$ and $\delta_j = 0$. Many*

candidates suggested that this individual both died and didn't die in the age year x to x+1. Others only explained one of the two variables, not both.

6. *Part (a)(ii) was done reasonably well. In (a)(ii) the question asked for the log-likelihood function for the data from a set of n lives. Some candidates lost points by using only one life, not all n lives.*
7. *Part (a)(iii) was done reasonably well by those who attempted it. For full credit, candidates were required to define* E_x^c *and* d_x *explicitly, as instructed in the question.*
8. *Part (a)(iv) was skipped by most candidates. Those who made a substantive attempt generally earned full points.*
9. *Part (b) was a gift to candidates. Almost all (who did not omit the question) earned the full point. A few lost a fraction of a point by only calculating the variance when the question asked for the standard deviation.*
10. *Part (c) was omitted by many candidates. Most candidates who attempted it used an intuitive heuristic, calculating confidence intervals for both estimates, and checking for overlap. This approach earned substantial partial credit. For full credit, candidates needed to use the (estimated) variance of the difference between the random variables to assess whether the difference is significantly different to zero.*
11. *Part (d) was done well by those who did not skip it.*

Question 5 (*12 points*)

(a) (*3 points*) (i) The 2-year transition matrix is (by matrix multiplication)

$$_2P_{80} = \begin{pmatrix} 0.80 & 0.15 & 0.05 \\ 0.40 & 0.40 & 0.20 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 0.70 & 0.20 & 0.10 \\ 0.30 & 0.40 & 0.30 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.605 & 0.220 & 0.175 \\ 0.400 & 0.240 & 0.360 \\ 0 & 0 & 1 \end{pmatrix}$$

(ii) Using the transition matrix above, the EPV of benefits is:

DB: $100,000\left({}_1p_{80}^{02}\ v + \left({}_2p_{80}^{02} - {}_1p_{80}^{02} \right) v^2 \right)$

$= 100,000\left[(0.05)(0.95) + (0.175 - 0.05)(0.95)^2 \right] = 16,031.25$

NHB: $50,000\left({}_1p_{80}^{01}\ v + {}_2p_{80}^{01}\ v^2 \right)$

$= 50,000\left(0.15(0.95) + (0.22)(0.95)^2 \right) = 17,052.50$

Total: $\mathbf{33,083.75}$

(b) (*1 point*) EPV Expenses: $500 + 300\left(1 + \left({}_1p_{80}^{00} + {}_1p_{80}^{01} \right) v \right) = 1,070.75$

$$\Rightarrow P = \frac{33,083.75 + 1,070.75}{1 + 0.8v} \quad = \mathbf{19,405.97}$$

(c) (*2 points*)

(i) ${}_1V^{(0)} = 50,000v\left({}_1p_{81}^{01} \right) + 100,000v\left({}_1p_{81}^{02} \right) + 300 - P$

$= 50,000(0.95)(0.20) + 100,000(0.95)(0.10) + 300 - 19.405.97$

$= \mathbf{-105.97}$

(ii) ${}_1V^{(1)} = 50,000v\left({}_1p_{81}^{11} \right) + 100,000v\left({}_1p_{81}^{12} \right) + 300$

$= 50,000(0.95)(0.4) + 100,000(0.95)(0.30)$

$= \mathbf{47,800}$

(d) (*5 points*) (i) $\Pr_1^{(0)} = (P - 300)(1.12) - p_{80}^{02}(100,000) - p_{80}^{01}\left(50,000 + {}_1V^{(1)}\right) - p_{80}^{00}\left({}_1V^{(0)} \right)$

$= (19,405.97 - 300)(1.12) - (0.05)(100,000)$

$- (0.15)(50,000 + 47,800) - (0.8)(-105.97)$

$= \mathbf{1813.46}$

(ii) $\Pr_2^{(0)} = \left({}_1V^{(0)} + P - 300 \right)(1.12) - p_{81}^{02}\left(100{,}000\right) - p_{81}^{01}\left(50{,}000\right)$

$= \left(-105.97 + 19{,}405.97 - 300 \right)(1.12) - (0.10)\left(100{,}000\right) - (0.20)\left(50{,}000\right) = \mathbf{1280}$

$\Pr_2^{(1)} = \left({}_1V^{(1)} - 300 \right)(1.12) - p_{81}^{12}\left(100{,}000\right) - p_{81}^{11}\left(50{,}000\right)$

$= \left(47{,}800 - 300 \right)(1.12) - (0.3)\left(100{,}000\right) - 0.4\left(50{,}000\right) = \mathbf{3200}$

(iii) $\Pr_0^{(0)} = -500$

$\Rightarrow$ Profit signature is

$$\Pi = \left(\Pr_0^{(0)}, \Pr_1^{(0)}, \left(p_{80}^{00}\ \Pr_2^{(0)} + p_{80}^{01}\ \Pr_2^{(1)} \right) \right)^{T} = \left(\mathbf{-500, 1813.46, 1504} \right)^{T}$$

(iv) NPV is $-500 + 1813.46 v_{10\%} + 1504\, v_{10\%}^{2} = 2391.58$

$$\Rightarrow \quad \text{PM is } \frac{2391.58}{P\left(1 + p_{80}^{00} v\right)} = \frac{2391.58}{19{,}405.97(1 + 0.8 v_{10\%})} = \mathbf{7.135\%}$$

(e) (*1 point*) Because the hurdle rate is less than the earned rate, so holding more capital is beneficial. We accumulate at 12% and discount at 10%.

Examiners' Comments

1. *Overall performance on this question was very variable. Very few candidates omitted the question entirely, and around 30% earned full credit on at least 3 parts of the question. However, for parts (b), (c), and (d), many candidates earned little or no partial credit.*
2. *Part (a) was done well overall. Many candidates used Excel to multiply the matrices in (a)(i), which earned full credit if correct. In (a)(ii) the most common error was using* ${}_2 p_{80}^{00}$ *for the probability of the death benefit payment in the second year, but that probability includes the probability that the life died in the first year. The deferred probability is required for the year 2 benefit values.*
3. *Part (b) was also well done, though many candidates lost some credit, and could not match the given value, because they assumed that the maintenance expenses are only incurred in the Healthy state.*
4. *Part (c) was generally well done, but again some candidates lost points by calculating net premium policy values instead of gross premiums. It is good practice in this exam to read the question several times as you work through it, to ensure that you have fully allowed for all of the information given in the question, including defining terms.*
5. *In part (d), performance was very polarized, likely separating candidates who had studied profit testing in the multiple state context from those who had not. A few candidates who correctly identified the profit signature then used the wrong interest rate for the NPV and profit margin.*
6. *Part (e) presented an unusual situation, where the hurdle rate was less than the earned rate on reserves, which inverted the usual relationship between NPV and reserves. A few candidates realized that in this case, it is beneficial to NED to hold more capital (i.e., reserves) at time 1.*

Question 6

(a) (2 points)

(i) Under Option A the annual rate of pension benefit at retirement is

$$FAS_{60} = 100{,}000\left(\frac{1.03^{9} + 1.03^{10}}{2}\right) = 132{,}434.5$$

$$B_{60} = FAS_{60} \times 0.015 \times 30 \times \left(1 - 5 \times 12 \times 0.003\right) = 48{,}868.32$$

$$\Rightarrow \text{ first month's pension is } \frac{48{,}868.32}{12} = \mathbf{4072.36}$$

(ii) Under Option B:

$$FAS_{65} = 100{,}000\left(\frac{1.03^{14} + 1.03^{15}}{2}\right) = 153{,}528$$

$$B_{65} = FAS_{65} \times 0.015 \times 35 = 80{,}602$$

$$\Rightarrow \text{ first month's pension is } \frac{80{,}602}{12} = \mathbf{6{,}716.84}$$

(b) (*1 point*) The assumption of no exits is valid if the cost to the plan of deaths or withdrawals is similar to, or less than the cost of retirements. That means that if early exits were explicitly allowed for, the resulting liability would be similar or less than the liability assuming all lives survive to retirement.

(c) (*3 points*) Note that the value at age x of a pension starting at a rate of B per year, payable monthly, and increasing monthly at a rate of 2% per year is:

$$\frac{B}{12}\left(1 + {}_{\frac{1}{12}}p_x (1.02v)^{\frac{1}{12}} + {}_{\frac{2}{12}}p_x (1.02v)^{\frac{2}{12}} + \cdots\right) = B\ddot{a}^{(12)}_{x|i^*}$$

where $i^* = \dfrac{1+i}{1.02} - 1 = \dfrac{1.06}{1.02} - 1 = 3.9216\%$

(i) Assuming Option A:

$$AL_A = FAS_{60} \times 0.015 \times 20 \times 0.82 \times v^{10} \times \ddot{a}^{(12)}_{60|i^*}$$

$$= (132{,}434.5)(0.015)(20)(0.82)(1.06)^{-10}(16.2437) \quad = \mathbf{295{,}503}$$

(ii) Assuming Option B:

$$AL_B = FAS_{65} \times 0.015 \times 20 \times v^{15} \times \ddot{a}^{(12)}_{65|i^*}$$

$$= (153{,}528)(0.015)(20)(1.06)^{-15}(14.5260) \quad = \mathbf{279{,}169}$$

(iii) The EPV of the accrued benefit for early retirement is significantly higher than the age 65 retirement (even though the age 65 value allows for 5 more years of salary increases) indicating that the reduction factor appears generous in this case.

OR

The proportionate reduction factor for age 60, based on actuarial equivalence, would be calculated as

$$\frac{v^5\ddot{a}_{65}^{(12)}}{\ddot{a}_{60}^{(12)}}=\frac{(1.06)^{-5}\times 14.5260}{16.2437}=0.6682389$$

and, as it is quite lower than $1-0.003\times 60=0.82$, the actuarial reduction factor offered by then plan seems too generous.

(d) (*2 points*)

(i) EPV of contributions = $0.08(100{,}000)(1.03)a_{\overline{1}|}^{(12)}$,

$$\text{where } a_{\overline{1}|}^{(12)}=\left(\frac{1-v}{12\left((1.06)^{1/12}-1\right)}\right)=0.969067$$

$\Rightarrow$ EPV $=0.08(100{,}000)(1.03)(0.969067) \quad =$ **7985.11**

(ii) NC gross is $AL_B\,/\,20=13{,}958.4$

Net NC rate is c s.t. $c(1.03)(100{,}000)a_{\overline{1}|}^{(12)}+7985.1=13{,}958.4$

$$\Rightarrow c=\frac{5973.3}{(1.03)(100{,}000)(0.969067)}=\mathbf{5.98\%}$$

(e) (*2 points*)

(i) The actuary should either model exits more precisely, based on data from the company itself, and other similar firms, or they should assume the safe-side assumption to ensure that contributions are sufficient if all employees select the more expensive option – in this case Option A.

(ii) The members who choose Option A may be those at higher income levels, who can afford to support their lifestyle with other sources of income, e.g. savings/inheritance. Higher income individuals tend to have lower mortality.

On the other hand, if early retirement is used largely by employees in failing health, e.g. in physically demanding jobs, then the mortality may be worse for early retirees than for those staying to age 65.

Examiners' Comments

1. *Overall, candidates found this question challenging, which is often the case for pension questions. Some marks were lost by candidates who answered a different question to the one asked, adding complexity. Candidates are advised that in this exam, they should re-read the question several times as they work through it, to ensure that they are answering the correct question with the correct information. This is particularly true for pension questions, where the questions tend to have a long list of information and assumptions, all of which will be relevant to the question.*

2. *Candidates are encouraged to use Excel for pension questions, which are often quite calculation-heavy. If you do use Excel, you should note in your answer booklet what calculations you have done in Excel, with enough detail for graders to be able to assess whether partial credit is merited if the answer is incorrect.*
3. *Part (a) was generally done well, apart from most common mistake in pension questions, which is allowing for 1 year too few or too many salary increases. Some candidates only used past service for this part of the question. When projecting final benefits, it is appropriate to use past and future service. We use past service when we are valuing the liability associated with the accrued benefits at the valuation date.*
4. *Part (b) was not done well. Many candidates suggested that the assumption would be valid of no members died or withdrew, which is true technically, but not relevant in a practical discussion of assumptions.*
5. *In part (c), many candidates wrongly valued the benefit based on past and future service – possibly because they had used both in part (a). In (c)(iii), for full credit, candidates were required to state and justify their opinion on whether the actuarial reduction factor is reasonable. No credit was given to answers that merely defined the actuarial reduction factor.*
6. *Most candidates omitted part (d), or attempted it but did not earn any points. Some of those who did attempt it valued all future contributions instead of just one year. Others ignored the monthly frequency, either because they did not know how to handle it, or because they did not notice that payments were monthly. As ignoring the frequency made this part very much easier, the partial credit available was small if payments in either or both parts were assumed to be annual. Only a small minority of candidates earned most or all of the points on this part.*
7. *Part (e) attracted a range of answers, some quite insightful. Most of those who did not skip this part earned at least 1 exam point, but very few earned full credit,*